Multi-Version Coding in Distributed Storage

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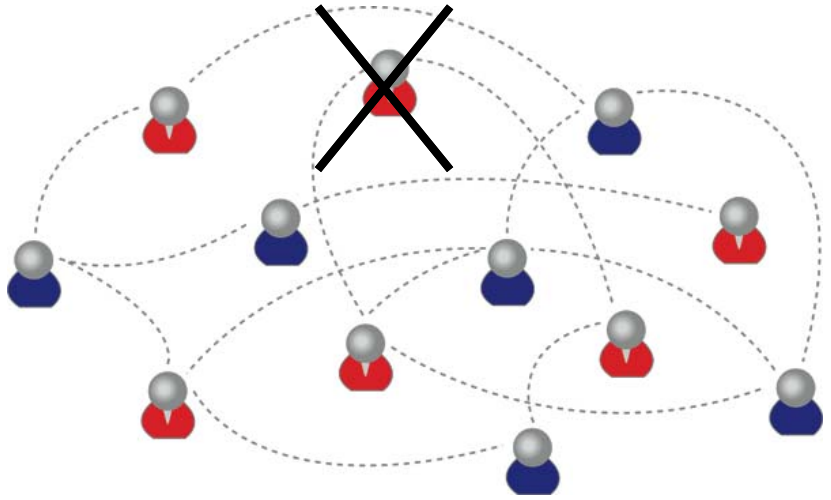
Coding in Distributed Storage

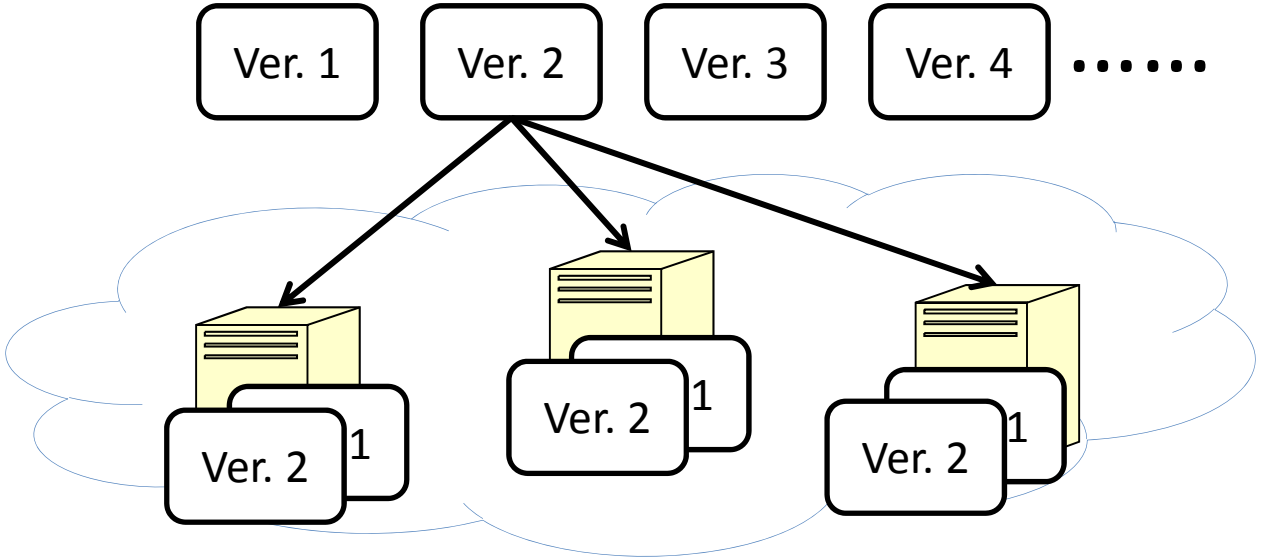
Coding in Distributed Storage

- Error-correction ability
- Redundancy
- Read/write, update, reconstruct
- Repair [Dimakis et al., 2007]
- …
- Assumptions:
  - One data set
  - Info available everywhere

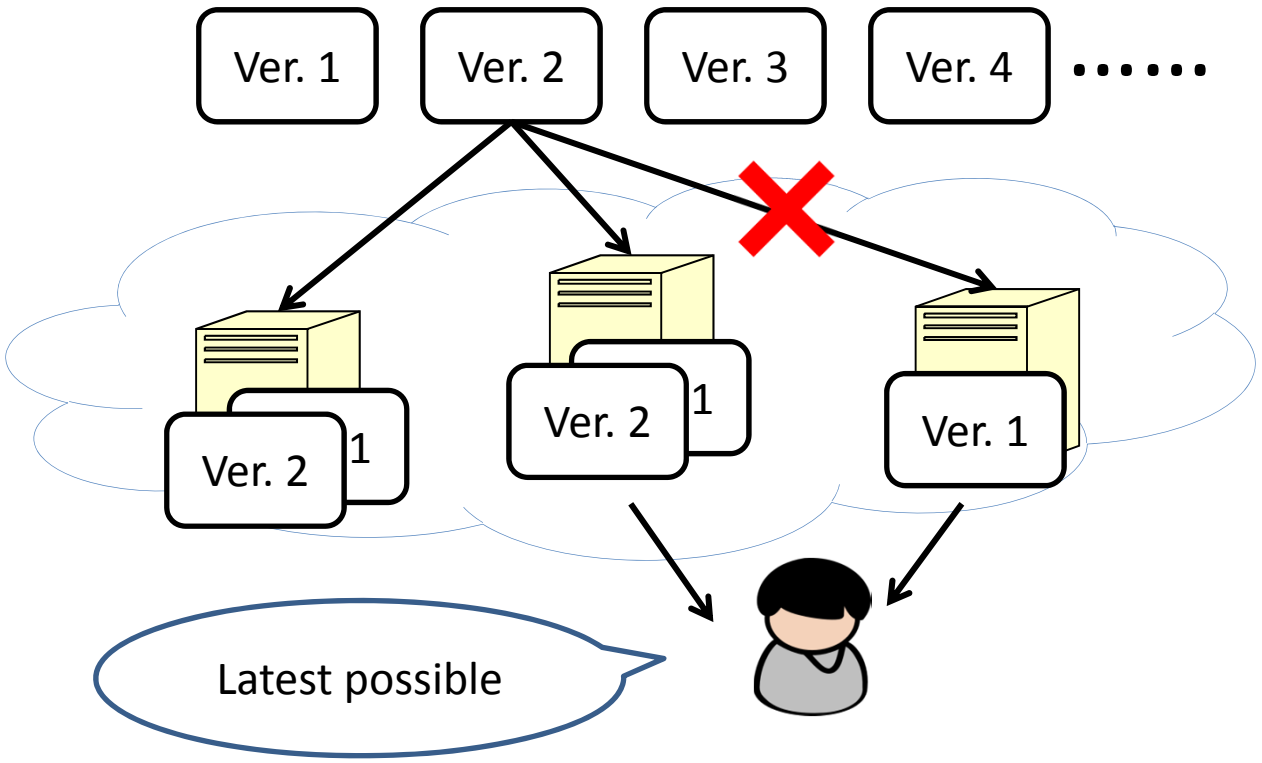


Data of Multiple Versions

- Later versions are more interesting

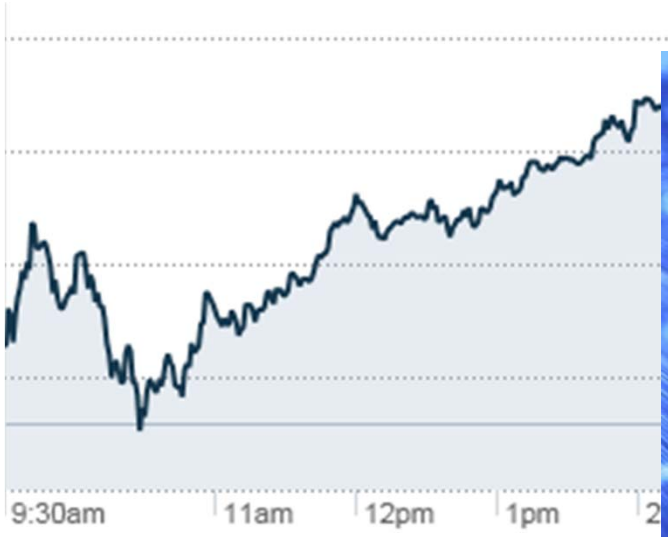


Snapshot in the Real World



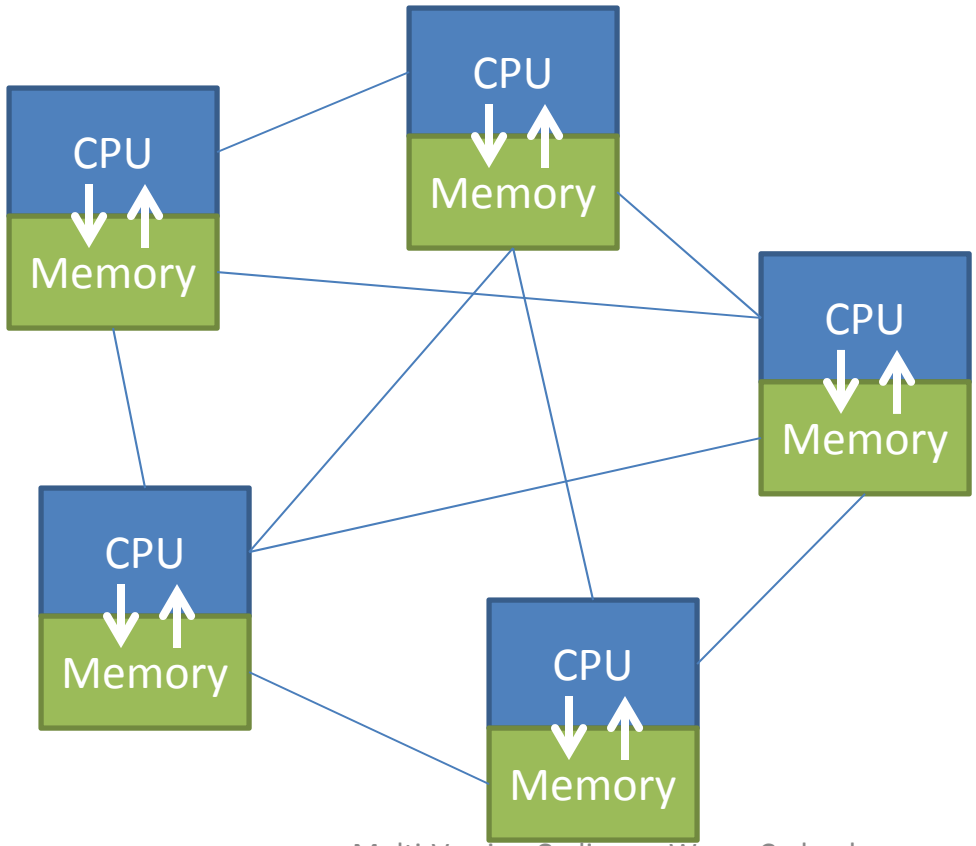
Why Interesting

- Information storage

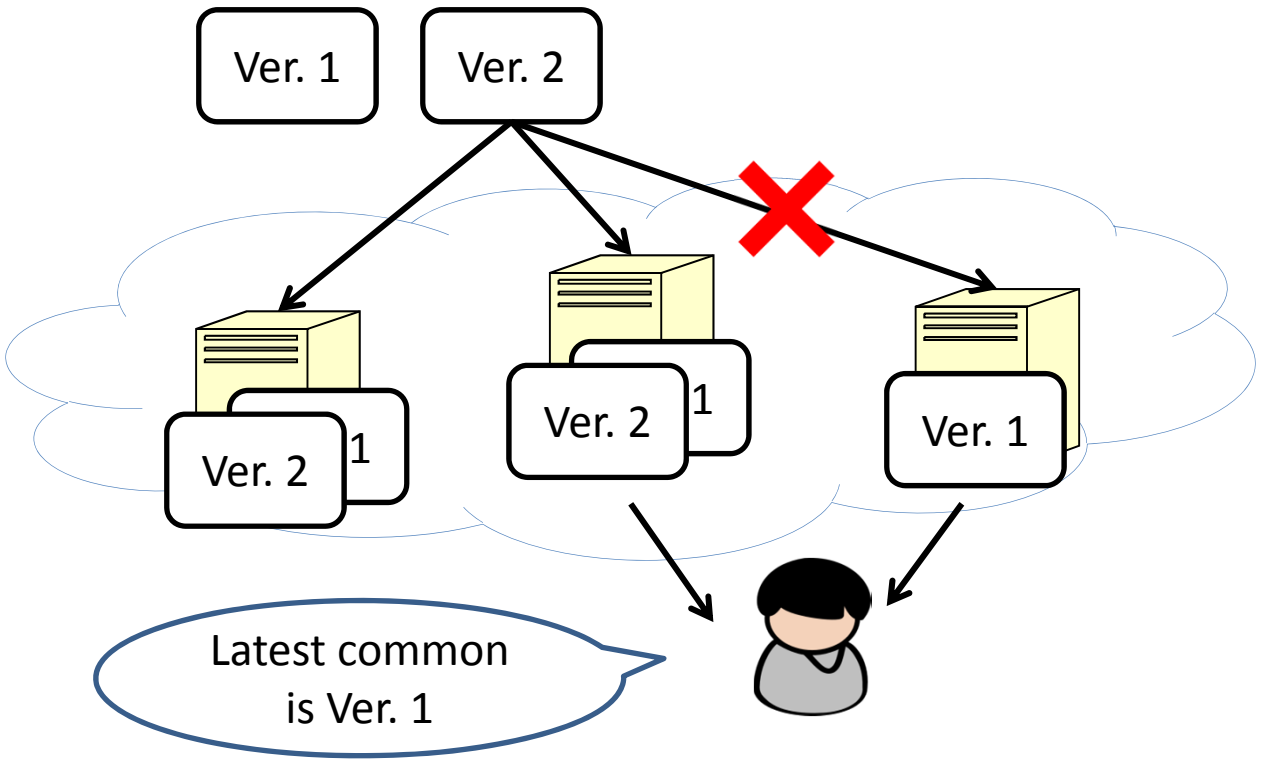


Why Interesting

- Multiprocessor programming



Our Problem: Multi-Version Coding



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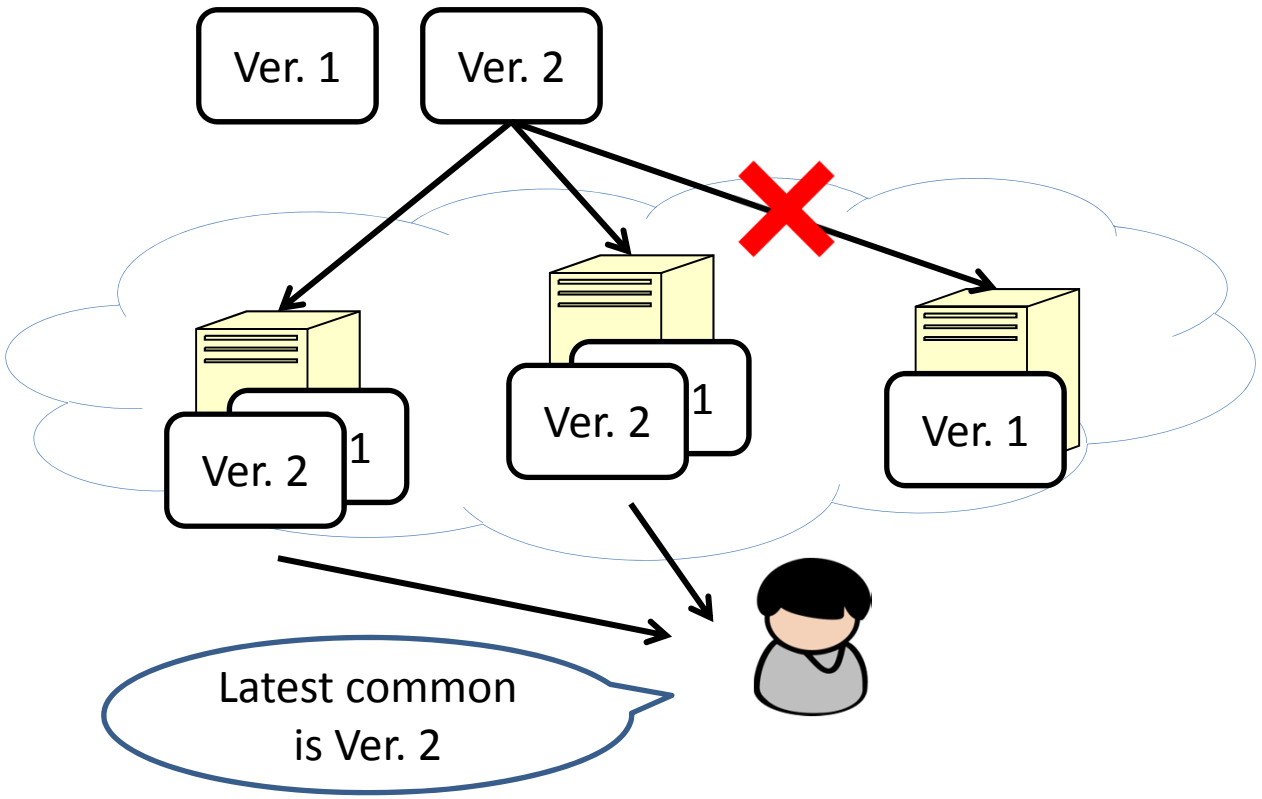
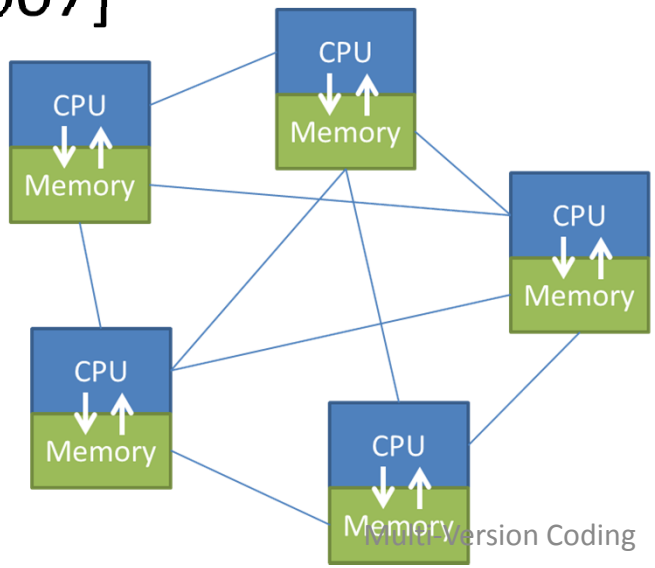


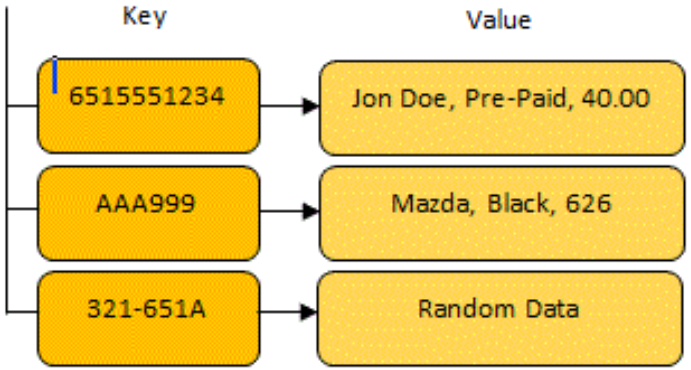
Features for Distributed Systems

- Consistency:
  - Decode the latest possible version
- Asynchrony
  - Every server receives different subsets of versions
  - Every server has no information about others

Connection to Distributed Computing

- Distributed computing theory
  - Shared memory system [Attiya et al., 1995]
- Practical distributed storage system
  - Amazon Dynamo key value store [DeCandia et al., 2007]



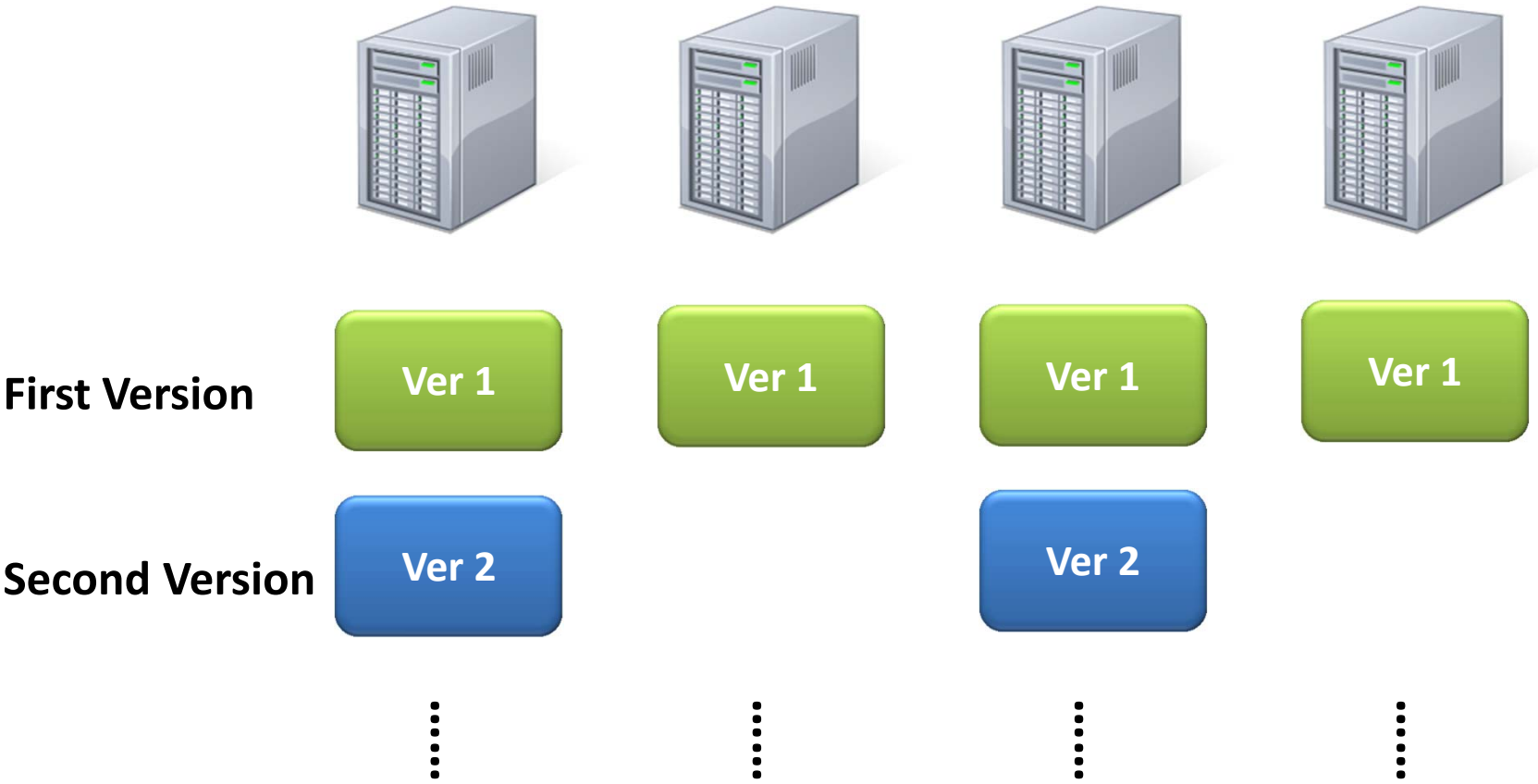


Problem Settings

- *n* servers
- *v* independent versions of the message
- *c* servers connected to a client
- Goal: recover the latest common version
- Q: worst-case storage size of each server?
  - Code construction
  - Bound

Replication

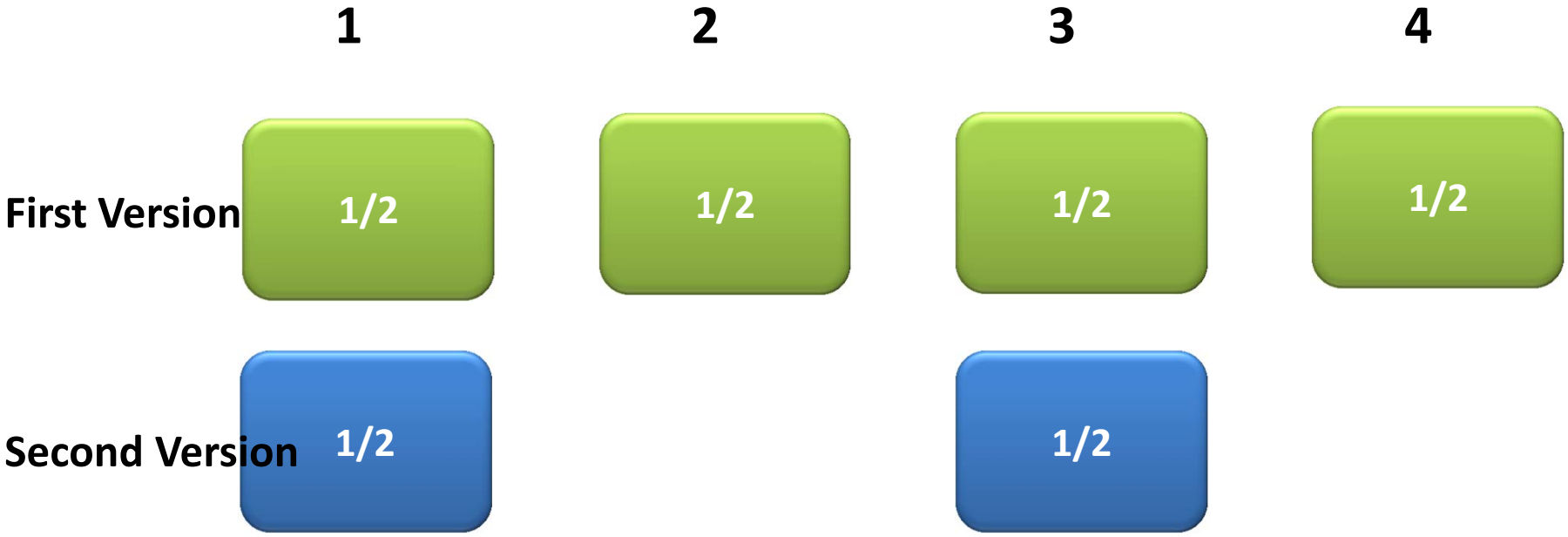
- Storage size = message size



[Attiya et al., 1995], [DeCandia et al., 2007]

(4,2) MDS Codes

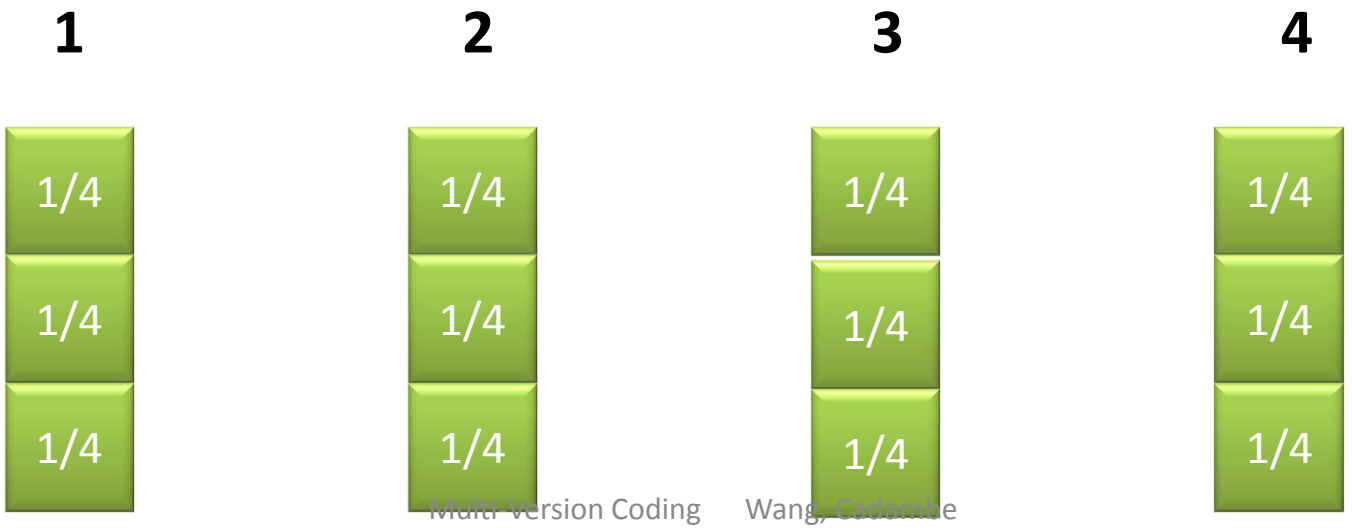
- Decode “latest common” from any 2 servers
- (Worst-cast) storage size = message size



[Hendricks et al., 2007], [Dutta et al., 2008], [Cadambe et al, 2013], [Dobre et al., 2013]

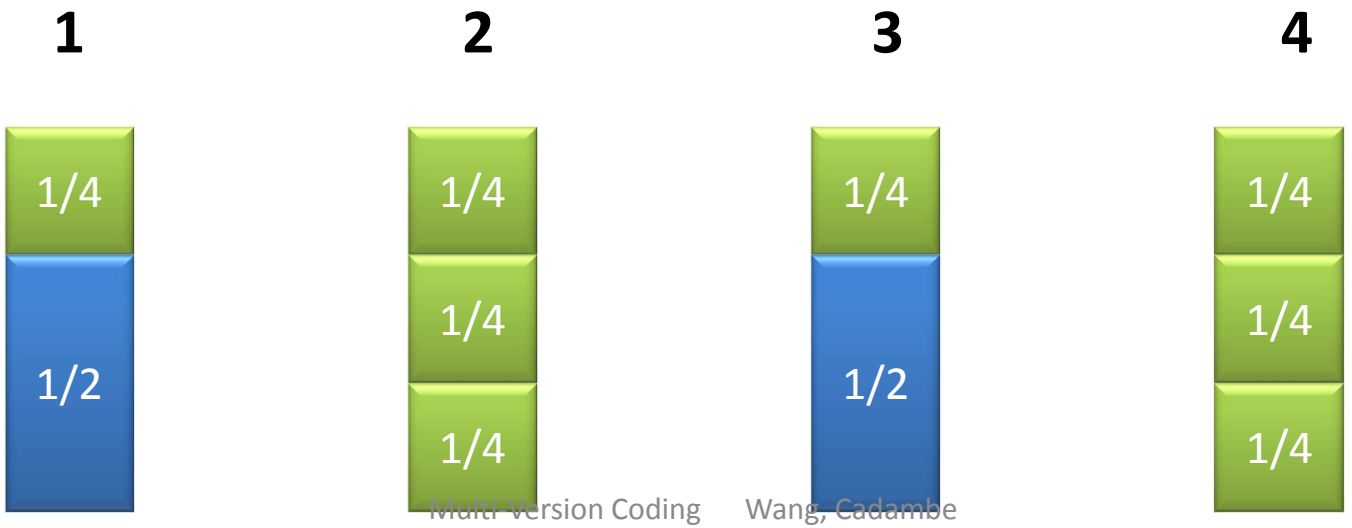
Our Code

- (9,4) code for Version 1



Our Code

- (9,4) code for Version 1
- (4,2) code for Version 2
- Decode “latest common” from any 2 servers
- Storage size = (message size)*3/4



Construction

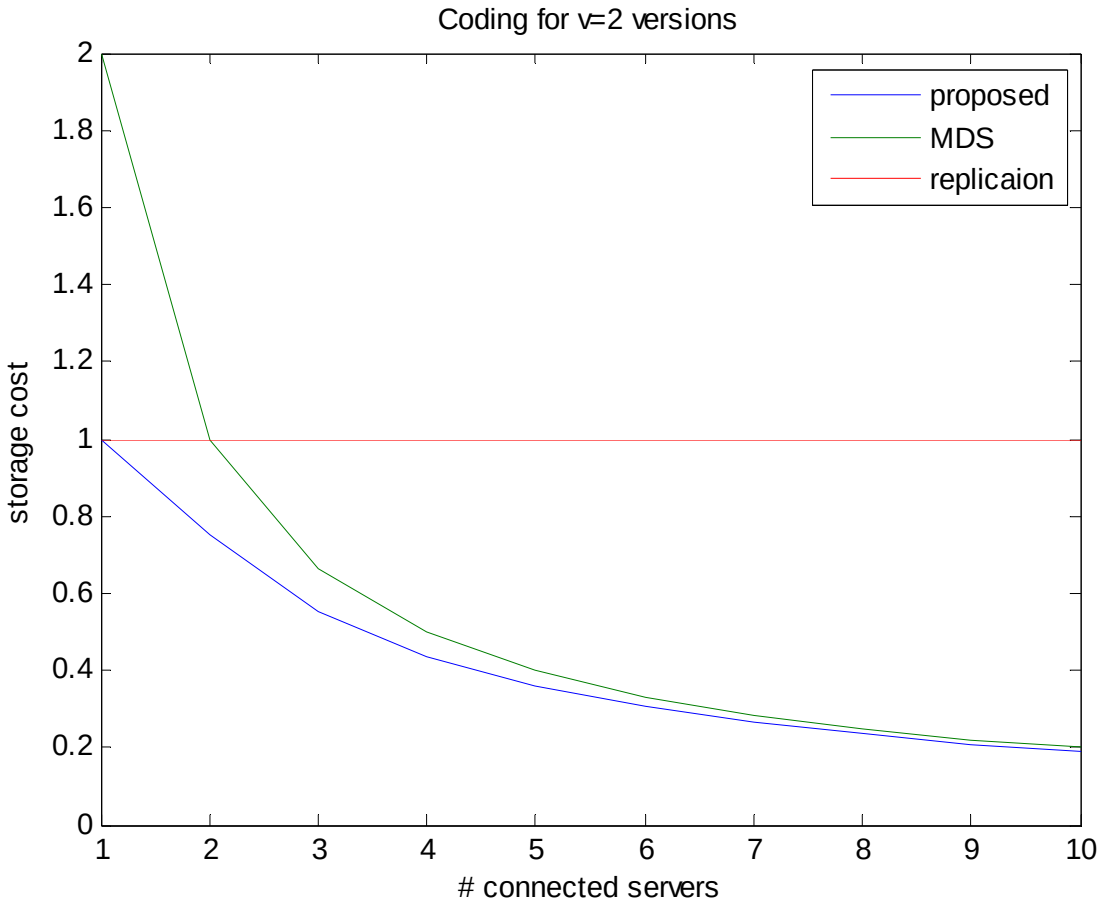
- Encode every version separately
- Assign different storage size for different versions
- Use MDS codes of different rates

- Find optimal assignment by linear programming
- Upon arrival of a new version, over-write

Construction

- Theorem: there exist a code for v versions and c connected servers such that
  - $\frac{\text{Storage size}}{\text{Message size}} = \frac{vc-v+1}{c^2}$
- Comparison
  - MDS: $\frac{v}{c}$
  - Replication: 1

Comparison


Coding for v=2 versions
proposed
MDS
replicaion
storage cost
connected servers
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
1
2
3
4
5
6
7
8
9
10

Bound

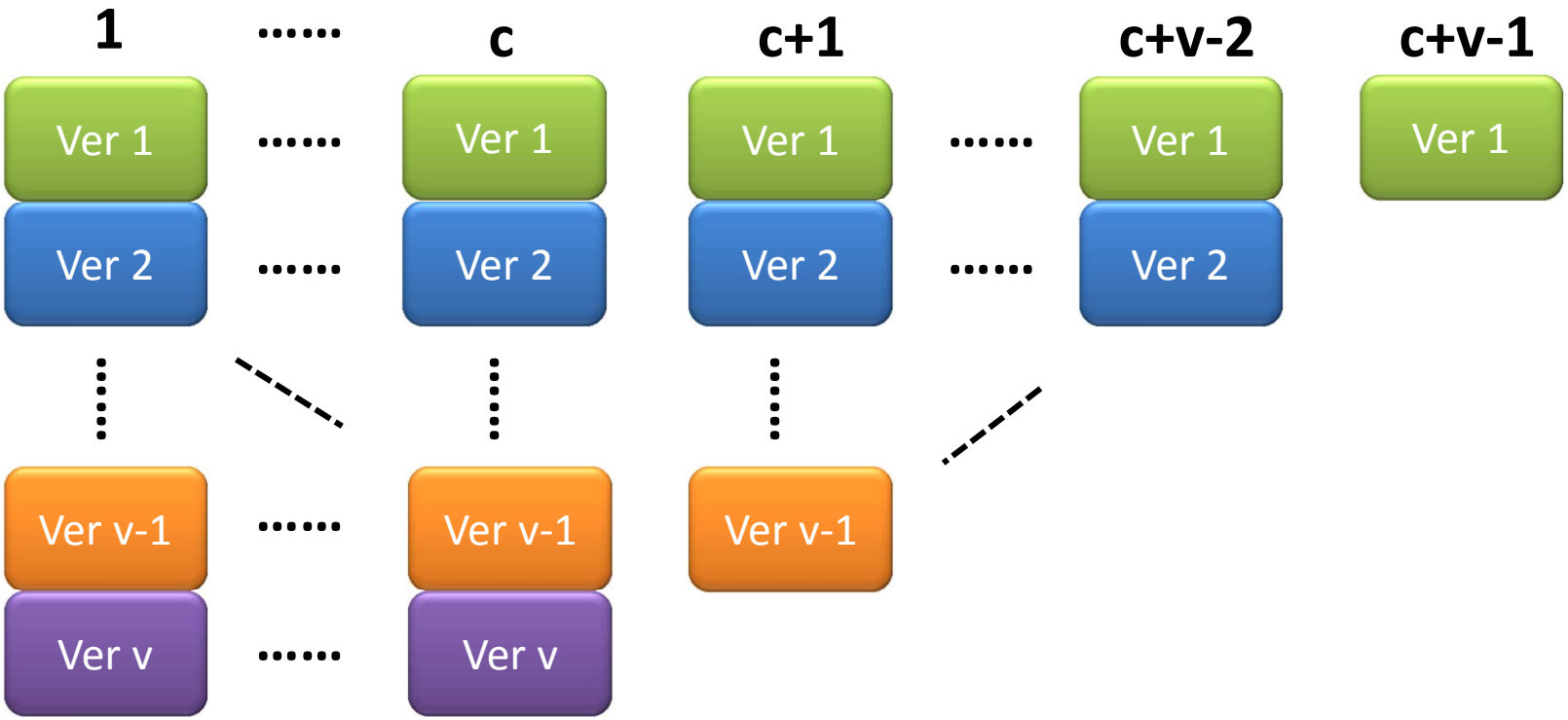
- Theorem: every multi-version code for v versions and c connected servers satisfies

$$\frac{\text{Storage size}}{\text{Message size}} \geq 1 - \left(1 - \frac{1}{c}\right)^{v}$$

- Tight for v=2 versions

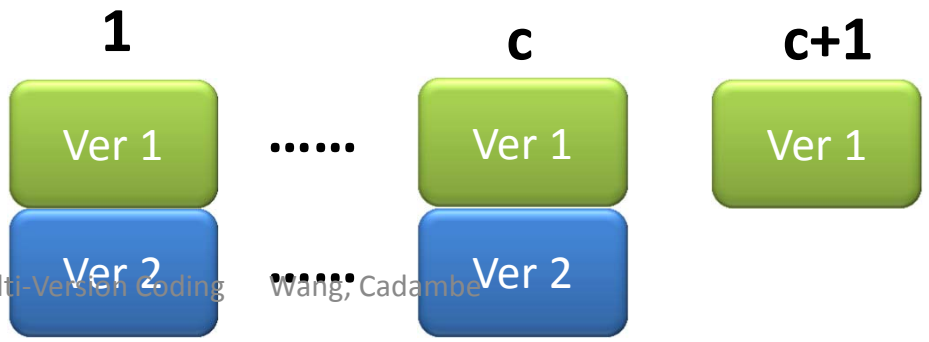
Proof Idea

- Consider a special scenario
- Lower bound the mutual information



Proof for v=2

- X_i information of Server i
- W_j information of Version j
- α server storage size
- B message size

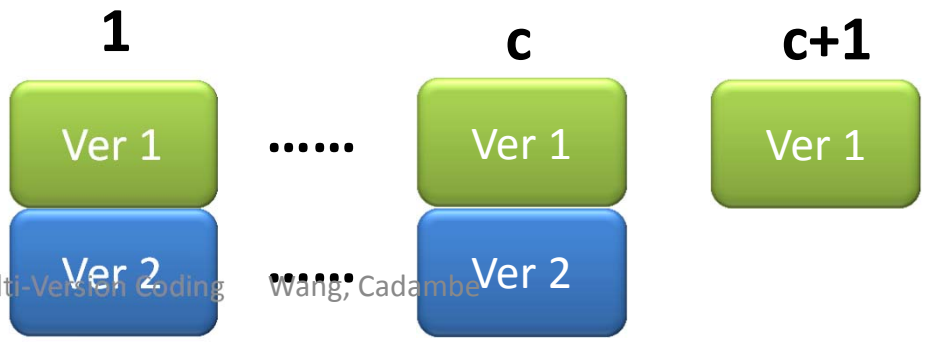


Proof for v=2

- Lemma 1. $I(X_{[c-1]}; W_1) \geq B - \alpha.$
- Lemma 2. W.l.o.g., $I(X_{[c-1]}; W_2|W_1) \geq \frac{c-1}{c} B.$
- Proof of theorem:

$$
\begin{aligned}
&(c-1)\alpha \\
\geq &I(X_{[c-1]}; W_1, W_2) \\
= &I(X_{[c-1]}; W_1) + I(X_{[c-1]}; W_2|W_1) \\
\geq &B - \alpha + \frac{c-1}{c} B.
\end{aligned}
$$

X_i information of Server i
W_j information of Version j
α server storage size
B message size



Future Directions

- Tight bound
- Dependent versions
- Average-case storage size
- Connect coding in distributed storage & distributed algorithms

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THANK YOU!