Estimation of Reproductive Number R_0

Jithin K. Sreedharan

April 16, 2020

"Stochastic" S.E.I.R. model

We assume a Susceptible-Exposed-Infected-Recovery (S.E.I.R.) compartmental epidemiological model.

- S_t, I_t, E_t and C_t are the number of people who are susceptible, infected, exposed, and cases. Recovered compartment is not expressed.
- D_e is the latent period. We assume $D_e = 4$ days .
- D_i is the infectious period. We assume $D_i = 8$ days.
- p is the case reporting rate.
- $beta(t)$ is the infection rate.
- z_t is the importations at time t.

"Stochastic" S.E.I.R. model (contd.)

It's not the true stochastic equations behind the deterministic S.E.I.R. model. Rather the following assume a prior distribution.

$$S_t = S_{t-1} - \beta(t) S_{t-1}(I_{t-1} + z_{t-1})\varepsilon_t \tag{1}$$

$$\varepsilon_t \sim \text{Lognormal}(\mu = 0, \sigma^2 = ?)$$

$$E_t = \beta(m, t) S_{t-1}(I_{t-1} + z_{t-1})\varepsilon_t + (1 - 1/D_e)E_{t-1} \tag{2}$$

$$I_t = E_{t-1}/D_e + (1 - 1/D_i)I_{t-1} \tag{3}$$

$$C_t \sim \text{Binom}(I_t, p) \tag{4}$$

$$\log(\beta(t)) := \theta_0 + \theta_1 m_t \tag{5}$$

The variable m_t in $\beta(t)$ denotes an intervention covariate (mobility, stay-at-home order).

A challenge in fitting the case data

- Underreporting: Number of reported cases tends to be less than the true number of infections due to asymptomatic or mildly symptomatic infections among people not seeking care or not being tested. The number of reported cases is captured in the parameter case reporting rate p.

Estimation of mean and variance of $\log \beta(t)$ for a given p

Let T be the date up to which data is available.
Data attributes: `date, daily_cases, daily_importations`.
Let c_t be the number of cases reported on day t.

1. Coarse estimate of I_t: c_t/p, for $1 \leq t \leq T$
2. Coarse estimate of E_t: $\frac{c_{t+D_e}}{p}\frac{D_e}{D_i}$, for $1 \leq t \leq T - D_e$
3. Smooth the coarse estimates using smoothing spline to get approximations for $\mathbb{E}[I_t|p]$ and $\text{cov}[I_t|p]$, and $\mathbb{E}[E_t|p]$ and $\text{cov}[E_t|p]$.
4. For the daily exposed cases ΔEt, calculate point estimates of $\mathbb{E}[\Delta E_t|p]$ and $\text{cov}[\Delta E_t|p]$
5. Point estimates of $\mathbb{E}[S_t|p]$ and $\text{cov}[S_t|p]$ from $\mathbb{E}[\Delta E_t|p]$ and $\text{cov}[\Delta E_t|p]$
6. Using eq. (2) of S.E.I.R. model, find point estimates of $\mathbb{E}[\log \beta(t)|p]$ and $\text{Var}[\log \beta(t)|p]$

Our methods are built on the following report published by Institute for Disease Modeling: "Social distancing and mobility reductions have reduced COVID-19 transmission in King County, WA"

Estimation of R_0

- With the estimated $\mathbb{E}[\log \beta(t)|p]$ and $\mathrm{Var}[\log \beta(t)|p]$, we construct $\mathbb{E}[\log \beta(t)]$ and $\mathrm{Var}[\log \beta(t)]$ via law of total expectation and law of total variance.
- Find the point estimate and confidence interval of the reproductive number R_0 using the expression

$$R_0(t) = \exp\{\log \beta(t)\} S_0 D_i \tag{6}$$

Work that is in pipeline

- Incorporating Facebook mobility data into the model
- Application of principles from Network Science
- Lifting constant rate assumption